Resources Policy 100 (2025) 105447

Contents lists available at ScienceDirect

Resources Policy

journal homepage: www.elsevier.com/locate/resourpol

Using metals to hedge carbon emission allowances – Tail-risk and Omega ratio analysis

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ARTICLE INFO

JEL classification:
G11
G32
L61

Keywords:
CO_2 allowances
Extreme risk
Parametric value-at-risk
The Omega ratio

ABSTRACT

In recent years, carbon allowances have experienced significant volatility as a mechanism for reducing CO2 emissions. This study constructs two five-asset portfolios that include carbon emission allowances and various metals, to evaluate which portfolio offers lower exposure to extreme risk and a more favourable return-to-risk profile. Extreme risk is assessed using several parametric VaR models, such as the traditional normal VaR, two non-normal models (logistic and hyper-secant), and the CVaR model. The Omega ratio is utilized to gauge performance in terms of return-to-risk. The portfolios are constructed for both pre-crisis and crisis periods. The similarities in the structure of the constructed VaR portfolios suggest that different objective functions have a limited impact on portfolio design. However, the selection of the VaR model does affect the estimated downside risk, which is crucial for the accuracy of the model and effective extreme risk assessment. Both portfolios function as effective hedges for carbon allowances, achieving a reduction in extreme risk of over 60% during both periods. Nevertheless, the precious metals portfolio, dominated by gold, outperforms the industrial metals portfolio. Analysis of the Omega ratio shows that the precious metals portfolio consistently provides better risk-adjusted returns at all threshold levels, indicating that investors can enhance their returns by combining carbon allowances with precious metals. This outperformance is largely attributed to the significantly lower risk of gold compared to other metal commodities. The results may provide essential guidance for investors and decision-makers alike.

1. Introduction

The greenhouse effect continues to be a pressing challenge for global sustainability in the modern era (Carraro and Favero, 2009; Katariya and Shukla, 2022; Wehner and Yu, 2023; Zhou et al., 2024).[1] Among the various strategies[2] to mitigate carbon emissions, carbon trading has emerged as an effective market-driven tool to achieve emissions reductions efficiently. A leading example of this approach is the European Union Emissions Trading System (EU ETS), the world largest carbon market, established in 2005 (Wei et al., 2021; Milanés-Montero et al., 2021). The EU ETS functions as a mandatory "cap-and-trade" framework (Berrisch et al., 2023). This system utilizes EU Allowances (EUAs) to permit holders to emit one metric ton of carbon dioxide (CO_2) or an equivalent volume of other greenhouse gases. Entities regulated under the EU ETS, such as industrial manufacturers, energy producers, and airlines, must secure enough EUAs to cover their yearly emissions (Huang et al., 2022). The system operates on a set emissions cap, which restricts the total greenhouse gases allowed from participating entities. This cap is gradually reduced over time, raising the cost of carbon emissions as scarcity of EUAs increases. Every February, these entities

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[1] The greenhouse effect is a natural phenomenon in which greenhouse gases retain heat within Earth atmosphere, helping to sustain temperatures necessary for life. Sunlight warms the Earth surface, which emits heat as infrared radiation. Greenhouse gases, like CO_2, absorb and re-radiate this heat, preventing it from escaping into space. Carbon dioxide is the main contributor to the enhanced greenhouse effect due to its high concentration from human activities like fossil fuel burning and deforestation, its long atmospheric lifespan, and its effectiveness in trapping heat.

[2] Other carbon pricing mechanisms are: 1) Carbon taxes directly charge a fee based on CO_2 emissions, 2) Carbon offsets enable the reduction of emissions through external projects, 3) Fuel taxes target the sale of fossil fuels, raising their costs to reduce consumption, 4) Carbon border adjustments ensure a level playing field by imposing tariffs on imports based on their carbon emissions, 5) Subsidies/Incentives support the adoption of clean technologies.

https://doi.org/10.1016/j.resourpol.2024.105447
Received 23 September 2024; Received in revised form 25 November 2024; Accepted 9 December 2024
Available online 17 December 2024

are allocated a certain number of EUAs (Batten et al., 2020), with each allowance representing the right to emit one ton of CO2-equivalent greenhouse gases (Ellerman et al., 2016). Organizations are required to submit a sufficient number of EUAs by April each year to compensate for their emissions from the previous year. (Dhamija et al., 2017). Organizations emitting below their allocated cap can trade their excess allowances, whereas those surpassing their limit must buy additional allowances or incur substantial fines. (Wang et al., 2022b). The trading of EUAs allows for flexibility, enabling companies to buy allowances when needed or sell them if they manage to reduce emissions. Covering 31 countries and accounting for almost half of the EU total carbon emissions, the EU ETS is responsible for over 70% of the global emissions trading value (Batten et al., 2020). Due to its cost-efficiency, extensive scope, and adaptability, the EU ETS has gained recognition as a successful policy for curbing CO2 emissions and fostering the development of low-carbon technologies (Wei et al., 2021).

However, the price of EUAs has become increasingly volatile as carbon trading activities intensify. As illustrated in Fig. 1, EUA prices have surged considerably since 2020, and several factors contribute to this upward trend. One key driver is the progressive reduction in the EU cap on greenhouse gas emissions under the EU Emissions Trading System. By limiting the number of available allowances over time, scarcity is created, pushing prices upward. This tightening of the cap, particularly during Phase 3 (2013–2020) and Phase 4 (2021–2030) of the ETS, has introduced stricter emissions limits. Additionally, economic growth within the EU has increased the demand for EUAs, especially in high-emission sectors such as steel, cement and power production. Companies in these industries, striving to meet more stringent emissions regulations, have been compelled to buy more allowances, further driving prices higher. Moreover, rising fossil fuel prices, such as for natural gas and coal, have also contributed to the escalation of carbon prices. In response to high fuel costs, power generators often shift to cheaper but more carbon-intensive fuels like coal, increasing the demand for EUAs to cover their additional emissions. Besides, the growing participation of financial investors has played a role in boosting EUA prices as well. With the carbon market becoming more liquid and transparent, speculators have entered the market, betting on future price increases and creating short-term demand spikes for allowances. As a result of these fluctuations, addressing the risks linked to volatile carbon prices has become a key focus for market participants (Jiao et al., 2018).

This paper explores the construction of multivariate portfolios by combining EUAs with precious and industrial metals. Metals are incorporated as supplementary assets due to their distinct behaviour compared to other market assets and their relatively low risk (Nasreen et al., 2024). In particular, the transmission channels that shape the connection between metals and EUAs include macroeconomic conditions, energy price fluctuations, regulatory policies, technological changes and investor behaviour. However, these factors often do not create a strong bond between metals and EUAs, resulting in a low correlation between the two assets, which makes metals a suitable choice for hedging EUAs. A potential low correlation between EUAs and metals provides a strong theoretical basis for combining these two assets in a portfolio. In the existing literature, some papers have constructed portfolios with EUAs (Reboredo, 2013; Balcilar et al., 2016; Wen et al., 2017; Gargallo et al., 2024), but none have attempted to hedge EUAs with metals. This is where we see an opportunity for our research.

The study aims to address two key objectives, reflecting the diverse interests of participants in the EUA market. First, it seeks to reduce the extreme risk associated with EUAs within the portfolio. Second, it strives to identify the optimal return-to-risk ratio for investors focused on maximizing returns per unit of risk. The traditional Modern portfolio theory of Markowitz (1952) uses the mean-variance procedure to minimize portfolio variance. The basic principles of this approach include diversification, risk management and the efficient frontier. On the other hand, the traditional Sharpe ratio (Sharpe, 1966) focuses on maximizing excess returns per unit of standard deviation. However, both approaches have limitations in risk assessment, as they equally weight both positive and negative returns.

The paper tries to minimize the downside risk of the portfolio, specifically focusing on extreme negative returns. To accomplish this, we apply parametric Value-at-Risk (VaR), which quantifies the worst possible loss a portfolio might face within a given confidence interval over a defined period (Pombo-Romero et al., 2024). VaR has been widely used to assess risk in carbon markets (e.g., Feng et al., 2012; Reboredo and Ugando, 2015; Abadie et al., 2017; Segnon et al., 2017). However, the traditional parametric VaR relies on the assumption of normal distribution, which is problematic due to the non-normal behaviour often observed in financial time series, such as fat tails and skewness. To mitigate this limitation, we take a novel approach by utilizing parametric VaR with two alternative fat-tailed probability distributions, logistic and hyperbolic secant, while using the standard normal VaR as a baseline comparison. Additionally, we incorporate conditional Value-at-Risk (CVaR) from Rockafellar and Uryasev (2002) into the portfolio framework, as VaR alone does not account for losses that exceed the threshold. CVaR helps manage the severity of losses beyond the VaR level.

Given the importance of accurately capturing extreme risk, errors in VaR estimation can lead to misguided investment choices by either over- or underestimating the level of risk (Chebbi and Hedhli, 2022). A thorough understanding of downside risk enables investors to make better-informed decisions and take proactive steps to reduce potential losses (Assaf, 2009). To ensure the precision of our risk models, we use the Kupiec (1995) standard coverage test to evaluate the effectiveness of various theoretical VaR models. This allows us to identify the model that most accurately predicts realized downside risk. All estimates are made at a 99% confidence level, indicating a 1% chance that actual losses will exceed the theoretical risk estimate.

From a return-to-risk perspective, the Sharpe ratio has a notable limitation in that it does not explicitly account for downside risk. To address this, Keating and Shadwick (2002) proposed the Omega ratio, a

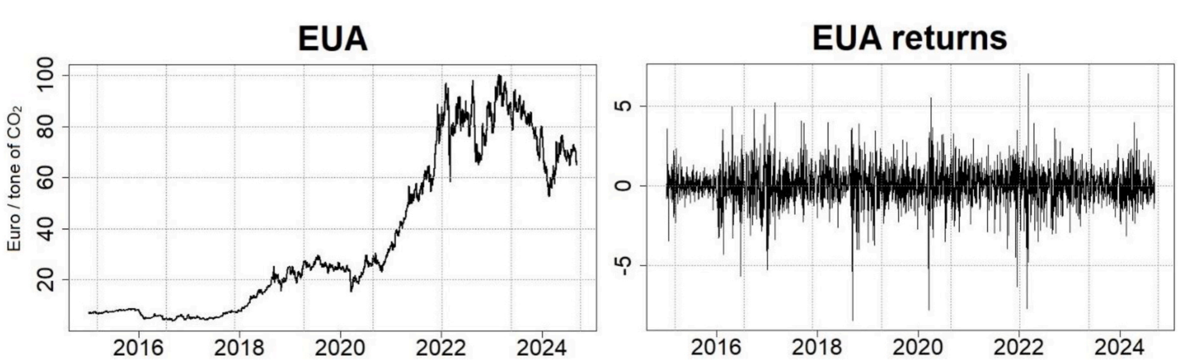

Fig. 1. Price dynamics of European Union Allowances.

more comprehensive performance measure. Unlike the Sharpe ratio, which primarily relies on the mean and variance of returns, the Omega ratio evaluates the entire probability distribution of an asset or portfolio. This broader approach significantly reduces the reliance on the assumption of normality (Bessler et al., 2021). Rather than isolating individual moments, the Omega ratio integrates the impact of all moments together, which is advantageous because it can be challenging to determine the relative importance of each moment.

A key difference from the Sharpe ratio is that the Omega ratio establishes a threshold value (τ), which separates gains from losses. Returns above this threshold indicate outperformance relative to investor expectations, while returns below it are considered losses. Yu et al. (2022a) highlight several advantages of the Omega ratio over traditional risk-adjusted performance metrics like the Sharpe, Sortino, and Treynor ratios. First, the Omega ratio allows investors the flexibility to select a threshold value that aligns with their specific risk and reward preferences. Second, it eliminates the need for assumptions regarding the return distribution, making it well-suited for non-normal distributions. Third, the Omega ratio simplifies portfolio management by eliminating the need for a covariance matrix, reducing the complexity of managing multi-asset portfolios. Finally, Keating and Shadwick (2002) claimed that the Omega ratio provides a practical edge over more intricate statistical metrics, particularly in estimating higher-order moments.

The study spans a substantial period of nine and a half years, which includes major events such as the COVID-19 pandemic and the war in Ukraine. It is likely that market conditions before these crises were significantly different from those during the crises. This allows us to divide the dataset into two distinct periods: pre-crisis and crisis periods. Following the methodology of Sikiru and Salisu (2023), the first subsample covers data up to December 31, 2019, representing the pre-crisis era, while the second subsample begins on January 1, 2020, capturing the crisis period. This segmentation enables us to examine how the portfolio composition and downside risk metrics behave across these two different phases.

This study offers several valuable insights that enrich the global body of research. First, it investigates the construction of two five-asset EUA portfolios that include both precious and industrial metals, with the goal of identifying which optimal portfolio minimizes extreme risk while maximizing risk-adjusted returns. A key innovation is the use of two unconventional VaR models within a multivariate portfolio optimization framework, an approach that has not been previously explored, to the best of our knowledge. This allows for the detection of subtle differences in extreme risk measurement, which is critical for informed investment decisions. Furthermore, the paper analyses how portfolio compositions shift across two distinct timeframes – before and during a crisis, which is in line with Vaissalo et al. (2024) who asserted that the crises might affect EUA prices.

Excluding the introduction, the structure of the paper is as follows: Section 2 surveys the relevant literature. Section 3 explains the applied methodologies, while Section 4 discusses the dataset utilized in the study. Section 5 focuses on the analysis of minimum VaR portfolios, and Section 6 examines the results pertaining to Omega portfolios. Section 8 provides an interpretation of the findings, and the concluding remarks are presented in the final section.

2. Literature review

This section reviews papers that utilized precious and industrial metals for portfolio construction. A significant number of papers investigate the hedging of oil with metals. For instance, Alomari et al. (2022) explore the relationship between quantile return spillovers and the interconnectedness of crude oil futures with major precious metals. From a portfolio construction perspective, precious metals serve as valuable diversification tools within oil-based portfolios. They asserted that precious metals generally offered greater hedging efficiency before the pandemic, with palladium standing out as the most effective hedge both before and during the pandemic. Mensi et al. (2020) explore the interactions, risk transfer dynamics, and portfolio impacts among major precious metals (gold, platinum, and silver) and energy commodities (crude oil, natural gas, gasoline, and gas oil) based on futures price returns. The analysis recommends that investors prioritize gold over silver and platinum when allocating resources alongside energy assets. Additionally, it reveals that hedge ratios are typically weak and highly responsive to market volatility in both energy and financial sectors. The study concludes that incorporating both energy commodities and precious metals into a portfolio offers better hedging benefits than focusing solely on energy assets, with gold proving to be the most reliable hedge during both stable and turbulent market conditions. Živkov et al. (2024) address extreme risk related to Brent oil by constructing multivariate portfolios that combine both precious and industrial metals within a multi-frequency framework. They assess extreme risk through parametric conditional Value-at-Risk (CVaR) and a more advanced semiparametric CVaR measure, using wavelet techniques to create portfolios across different time horizons. Their results indicate that gold consistently offers the most effective risk reduction, especially in oil-heavy portfolios, delivering the lowest risk and the best performance in back-testing and forecasting. Mensi et al. (2021b) investigate the relationships among 28 commodity futures markets, spanning precious and industrial metals, energy, agriculture, and livestock. Their portfolio risk analysis suggests that including WTI crude oil alongside other commodities enhances downside risk protection, particularly in the short term compared to the long run.

Other papers try to hedge stocks with metals. For example, Lei et al. (2023) investigate the role of precious metals as safe-haven assets for global Environmental, Social, and Governance (ESG) stocks, employing cross-quantilogram analysis and the quantile time-frequency connectedness framework. Their results indicate that palladium offers short-term safe-haven protection for ESG markets in North America, Europe, and developed Asia-Pacific regions, while gold serves as a less robust safe haven. This safe-haven effect persists through the COVID-19 period. Furthermore, they highlight that gold is effective for diversifying portfolios and reducing downside risks. Ali et al. (2021) reassess the contribution of gold, silver, and platinum to the diversification of six Dow Jones Islamic (DJI) equity index portfolios. Using various techniques, such as dynamic conditional correlations (DCCs), four-moment modified VaR, CVaR, and the global minimum-variance (GMV) portfolio strategy, their analysis reveals that constructing a GMV portfolio in the post-COVID-19 period necessitates a higher allocation to DJI Japanese equities and gold. Al-Nassar et al. (2023) investigate the role of alternative investment assets – gold, Bitcoin, oil, and the oil price volatility index (OVX) – as hedging tools and safe havens against risks in the Saudi stock market and its sectors during various phases of the COVID-19 pandemic. By applying the bivariate DCC-GARCH approach to capture volatilities and conditional correlations, the study reveals that gold consistently outperformed other assets in terms of optimal portfolio weights, with its prominence reaching a maximum during the pandemic period. Al-Nassar et al. (2024) explore the return and volatility spillover effects and their implications for portfolio management across regional stock super-sectors, as well as mineral (Energy, Industrial, and Precious Metals) and renewable (Agricultural and Livestock) commodities. In terms of portfolio strategy, they argue that the optimal portfolio allocations demonstrate two distinct periods of "flight to safety" towards commodities – excluding Energy –driven by the pandemic and the war, as investors reduced their stock holdings and increased their investments in commodities.

Some studies explore the use of precious metals to hedge against risks in industrial metals. In particular, Mensi et al. (2021a) analyse the dynamics between precious and industrial metals futures under various market conditions and investment horizons. From a portfolio perspective, they suggest that incorporating gold into industrial metal portfolios is beneficial for reducing risk without sacrificing expected returns, regardless of the investment horizon.

Relatively few papers, such as Sakemoto (2018), examine whether metals can act as hedges or safe havens for currency investment portfolios. He utilizes three key strategies: carry, momentum, and value. The findings indicate that both gold and silver offer hedging and safe-haven advantages across all strategies, with silver providing significant protection during periods of extreme market turbulence. However, these benefits diminished after 2000. In contrast, industrial metals were found to be ineffective as hedges or safe havens, particularly within carry portfolios.

3. Methodologies

3.1. Parametric VaR models

The paper uses a strategy for constructing multi-asset portfolios that aim to minimize Value-at-Risk (VaR) through a parametric method, utilizing different mathematical models to compute VaR. This approach diverges from the classic mean-variance optimization proposed by Markowitz (1952) by replacing variance with parametric VaR as the optimization criterion. The process begins with the variance-covariance matrix, as described in Equation (1), to identify the portfolio that achieves the lowest variance. In constructing the portfolio, a constraint ensures that the sum of all asset weights equals one, with individual asset weights constrained between zero and one (Equation (2)). The mean return of the minimum-variance portfolio is calculated as the weighted average of the asset returns, where ($\overline{\mu}_p$) represents the portfolio mean return and (μ_i) denotes the return of each individual asset (Equation (3)).

$$min\ \sigma_p^2 = min \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j} \tag{1}$$

$$\sum_{i=1}^{N} w_i = 1; \quad 0 \le w_i \le 1 \tag{2}$$

$$\overline{\mu}_p = \sum_{i=1}^{N} w_i \mu_i \tag{3}$$

in Equation (1), σ_p^2 represents the portfolio variance, while σ_i^2 denotes the variance of an individual asset i. w_i signifies the weight allocated to asset i, and $\rho_{i,j}$ refers to the correlation coefficient between any two assets, i and j.

The traditional parametric VaR method operates under the assumption of normality (VaR^{norm}) and is calculated using the portfolio mean return ($\overline{\mu}_p$) and standard deviation (σ_p) as defined in Equations (3) and (1): $VaR_p^{norm} = \overline{\mu}_p + Z_\alpha \sigma_p$. Here, $\overline{\mu}_p$ represents the central tendency (location parameter), and σ_p reflects the variability (scale parameter) of the distribution. The term Z_α is the left quantile of the inverse normal distribution, while α represents the chosen confidence level, set at 99% in this scenario. This indicates that only severe negative returns, located in the extreme left tail of the distribution, are considered.

The normal VaR model is often seen as too simplistic, as daily commodity time-series typically deviate from the normal distribution. To address this shortcoming, our research explores two alternative probability density functions with heavier tails, logistic and hyper-secant, to identify which distribution better captures the behaviour of empirical data. These alternative distributions offer a higher probability of extreme one-day events compared to the normal model. Specifically, the kurtosis for the logistic and hyper-secant distributions is 1.2 and 2, respectively, while the normal distribution has a kurtosis of zero. These non-normal distributions are particularly advantageous for VaR estimation due to their invertible property, ensuring a one-to-one correspondence between inputs and outputs, which facilitates precise mappings. To compute VaR with these distributions, we apply specific transformations to derive their inverse functions.

Equation (4) outlines the mathematical expression for the logistic distribution. By rearranging the equation to express x as a function of $F(x)$, the inverse function can be derived. The transformation of the logistic function is present in Equation (5). In this context, $F(x)$ denotes the quantile function, and the inverse of the logistic distribution is then used to compute the logistic Value-at-Risk (VaR), as shown in Equation (6).

$$F(x) = \frac{1}{1 + e^{-\frac{x-\mu}{a}}} \tag{4}$$

$$F(x) = \frac{1}{1 + e^{-\frac{x-\mu}{a}}} \Rightarrow \frac{1}{F(x)} - 1 = e^{-\frac{x-\mu}{a}} \Rightarrow \frac{1-F(x)}{F(x)} = e^{-\frac{x-\mu}{a}} \Rightarrow \tag{5}$$

$$\frac{F(x)}{1-F(x)} = e^{\frac{x-\mu}{a}} \Rightarrow ln\left(\frac{F(x)}{1-F(x)}\right) = \frac{x-\mu}{a} \Rightarrow x = \mu + a ln\left(\frac{F(x)}{1-F(x)}\right)$$

$$VaR_p^{log} = \overline{\mu} + a \times ln\left(\frac{F(x)}{1-F(x)}\right) \tag{6}$$

The location parameter is presented by the mean ($\overline{\mu}$), and the scale parameter (a) is determined using the formula $a = \sqrt{3\sigma^2/\pi^2}$, in the logistic distribution.

Equation (7) defines the hyper-secant distribution, and derivation of corresponding inverse function is shown in Equation (8). The calculation of the hyper-secant Value-at-Risk (VaR) is presented in Equation (9).

$$F(x) = \frac{2}{\pi}\arctan\left[e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}\right] \tag{7}$$

$$F(x) = \frac{2}{\pi}\arctan\left[e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}\right] \Rightarrow \frac{\pi}{2}F(x) = arctan\left(e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)}\right) \Rightarrow \tag{8}$$

$$tan\left(\frac{\pi}{2}F(x)\right) = e^{\frac{\pi}{2}\left(\frac{x-\mu}{\sigma}\right)} \Rightarrow ln\left(tan\left(\frac{\pi}{2}F(x)\right)\right) = \frac{\pi}{2}\frac{x-\mu}{\sigma} \Rightarrow x = \mu + \frac{2\sigma}{\pi} ln\left(tan\left(\frac{\pi}{2}F(x)\right)\right)$$

$$VaR_p^{hyps} = \overline{\mu} + \frac{2\sigma}{\pi} ln\left(tan\left(\frac{\pi}{2}F(x)\right)\right) \tag{9}$$

The hyperbolic secant Value-at-Risk (VaR^{hyps}) is computed in a manner similar to the normal distribution VaR, relying on the portfolio mean and standard deviation for the calculation.

In addition to parametric VaR portfolios, we also estimate parametric conditional VaR portfolio, which accounts for losses exceeding the VaR threshold. CVaR is calculated by integrating over the VaR, as shown in Equation (10).

$$CVaR_\alpha = -\frac{1}{\alpha}\int_0^\alpha VaR(x)dx \tag{10}$$

To measure the reduction in downside risk of EUAs within metal portfolios, we compute Hedge Effectiveness Indices (HEI). Accordingly, considering a specific risk measure (RM), HEI_{RM} can be calculated as follows:

$$HEI_{RM} = \frac{RM_{EUA} - RM_{portfolio}}{RM_{EUA}} \tag{11}$$

3.2. Kupiec back-test

To identify which VaR model most accurately reflects the empirical returns, we utilize the Kupiec (1995) standard coverage test. This test assesses the likelihood of observing a loss that surpasses the predicted VaR level, effectively determining whether the model failure rate matches the expected rate. To conduct the test, it is necessary to first establish the following exception indicator (I_t):

$$I_t = \begin{cases} 1, if\ r_{t+1} < VaR_\alpha \\ 0, if\ r_{t+1} \geq VaR_\alpha \end{cases} \tag{12}$$

Following Orhan and Köksal (2012), $\mathbb{F} = \sum I_t$ represents the total number of violations over N days. This test uses a straightforward approach, applying the binomial distribution to calculate the probabilities of failures, with $\mathbb{F} \sim B(N, \alpha)$. Under the null hypothesis, the failure ratio, $\frac{\mathbb{F}}{N}$, should equal α, or H0: $\frac{\mathbb{F}}{N} = \alpha$. The Kupiec test is carried out using a likelihood ratio (LR) approach, with the test statistic presented as in equation (13):

$$LR = 2\ ln\left(\left(1 - \frac{\mathbb{F}}{N}\right)^{N-\mathbb{F}} \left(\frac{\mathbb{F}}{N}\right)^{\mathbb{F}}\right) - 2\ ln\left((1-\alpha)^{N-\mathbb{F}}(\alpha)^{\mathbb{F}}\right) \tag{13}$$

The LR statistic follows a χ^2 distribution with 1 degree of freedom. Under the null hypothesis, H0: $\frac{\mathbb{F}}{N} = \alpha$, the test statistic yields a value of 0, which increases as the ratio of $\frac{\mathbb{F}}{N}$ deviates further from α. In other words, a risk model is deemed invalid if it produces an excessive or insufficient number of violations.

3.3. Omega ratio

Traditional return-to-risk metrics like the Sharpe ratio are limited in that they only consider mean and standard deviation, failing to account for the full characteristics of empirical distributions. This method assumes normality, overlooking the influence of higher moments such as skewness and kurtosis. Keating and Shadwick (2002) proposed the Omega ratio to overcome these shortcomings by dividing returns into two parts: those exceeding and those falling below a chosen threshold. Unlike the Sharpe ratio, the Omega ratio retains all the features of the time series data, allowing it to be calculated from historical returns and providing a more comprehensive performance assessment beyond just mean and variance (Kane et al., 2009).

The Omega ratio, as defined in Equation (14), uses $F(x)$ to represent the cumulative probability distribution, with τ indicating the investor-selected threshold value, and a and b specifying the upper and lower investment intervals. Essentially, the Omega ratio compares the probability-weighted gains to the probability-weighted losses relative to the chosen threshold (τ).

$$\Omega(\tau) = \frac{\int_\tau^a (1 - F(x))dx}{\int_b^\tau F(x)dx} \tag{14}$$

Avouyi-Dovi et al. (2004) argue that the Omega ratio does not rely on assumptions about risk preferences or utility functions. Instead, it only requires the specification of a threshold value as a straightforward decision rule. This simplifies decision-making, as more money is always preferable to less, making assets with a higher Omega ratio more attractive than those with a lower ratio. Vilkancas (2014, 2016) further suggests that evaluating the Omega ratio at various threshold values (τ) offers a more nuanced view of an asset or portfolio performance. Accordingly, we compute the Omega ratio for five different thresholds, creating the Omega function. The daily thresholds used are 0%, 0.002%, 0.004%, 0.006%, and 0.008%, which correspond to annual returns of 0%, 0.654%, 1.734%, 3.515%, and 6.448%, respectively. These thresholds are applied uniformly for both individual asset analysis and portfolio optimization. All calculated Omega functions display a downward trend, indicating that higher thresholds lower the probability of achieving substantial returns.

In optimizing portfolios using the Omega ratio, we adopt a non-parametric linear model as recommended by Mausser et al. (2006) and Yu et al. (2022b). This approach avoids assumptions about the return distribution by relying on historical data and corresponding sample measures. Unlike traditional mean-variance optimization, which directly employs first and second moments along with the covariance matrix, Omega ratio optimization does not follow this method.

According to Yu et al. (2022b), Omega ratio optimization can be expressed as a linear model:

$$\text{Max } \Omega \tag{15}$$

$$\text{subject to} : \delta\left(\sum_{i=1}^{n} w_i\mu_i - \tau\right) - (1-\delta)\frac{1}{T}\sum_{t=1}^{T}\eta_t \geq \Omega \tag{16}$$

$$\eta_t \geq \tau - \sum_{i=1}^{n}\mu_{it}w_i, t = 1, 2, \ldots, T \tag{17}$$

$$\eta_t \geq 0, t = 1, 2, \ldots, T \tag{18}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{19}$$

$$\sum_{i=1}^{n} w_i\mu_i \geq \tau \tag{20}$$

$$w_i \geq 0, i = 1, 2, \ldots, n \tag{21}$$

The objective function is designed to maximize the Omega ratio, which measures the deviation between portfolio returns and losses. μ_i represents the average return of the selected assets (i), and w_i denotes the weight of commodity (i) in the Omega portfolio. The parameter δ controls the balance between return and risk, while τ denotes the threshold for portfolio returns. The first term of Equation (16), $\left(\sum_{i=1}^{n} w_i\mu_i - \tau\right)$, captures the portfolio excess gain above the threshold. The second term, $\frac{1}{T}\sum_{t=1}^{T}\eta_t$, reflects the portfolio loss. Equations (17) and (18) are used to measure periodic losses. For portfolio optimization, it is essential that the sum of all asset weights equals one (Equation (19)). Additionally, the portfolio return must meet or exceed the threshold level defined by the required return (Equation (20)). Equation (21) ensures that all weights are non-negative, prohibiting short selling.

4. Dataset

This paper employs daily futures data for EUAs, along with precious metals (gold, silver, platinum, and palladium) and industrial metals (aluminum, copper, lead, and zinc). The dataset spans a substantial period, from January 2015 through August 2024, which allows for a clear comparison of pre-crisis and crisis portfolios. The division between the two periods is marked by January 1, 2020. Data are sourced from the investing.com platform, and futures prices are converted into log-returns ($\mu_{i,t}$) using the formula $\mu_{i,t} = 100 \times log(P_{i,t}/P_{i,t-1})$, where P_i denotes the commodity price. To ensure consistency across categories, all assets are synchronized for accurate comparison of individual asset and portfolio Omegas. EUA is paired separately with each group of metals, resulting in a slight variation in the number of observations for the two portfolios after synchronizing the data. Consequently, there are small discrepancies in the EUA statistics between the two portfolios (refer to Table 1). Table 1 provides descriptive statistics for the assets, highlighting the first four moments across the two distinct periods.

According to Hair et al. (2009), data analysis is important in conducting multivariate portfolio optimization. In particular, Table 1 reveals that all commodities exhibit relatively high kurtosis, suggesting an elevated level of downside risk. This indicates that employing fatter-tailed VaR functions and the CVaR model may be more suitable than traditional VaR approach. EUA stands out as having the greatest risk compared to the metals, implying that metals could serve as strong diversifiers when included in a portfolio with EUA. In terms of the Omega ratio, the Table offers insights into which commodity may hold

Table 1
Descriptive statistics of the selected commodities in the two sub-periods.

	Pre-crisis period				Crisis period			
	Mean	St. dev.	Skewness	Kurtosis	Mean	St. dev.	Skewness	Kurtosis
Panel A: Precious metals portfolio								
EUA	0.046	1.217	−0.236	6.530	0.033	1.229	−0.438	7.712
Gold	0.008	0.350	0.178	5.923	0.018	0.450	−0.298	6.507
Silver	0.003	0.600	−0.381	6.684	0.020	0.920	−0.450	7.416
Platinum	−0.011	0.515	−0.100	4.049	−0.002	0.875	−0.267	6.416
Palladium	0.029	0.725	−0.293	4.614	−0.027	1.214	−0.157	10.124
Panel B: Industrial metals portfolio								
EUA	0.039	1.222	−0.222	6.497	0.033	1.234	−0.439	7.714
Aluminium	0.000	0.480	0.186	5.657	0.009	0.613	−0.012	4.750
Copper	−0.003	0.540	0.040	4.359	0.008	0.627	−0.219	4.445
Lead	0.001	0.615	0.180	4.536	0.001	0.603	0.063	4.657
Zinc	0.002	0.661	0.217	4.872	0.007	0.713	−0.086	4.070

the highest value. The mean plays a crucial role, as it reflects potential for outperformance, with EUA likely having the highest Omega ratio. The asset characteristics vary significantly, particularly across the different sub-periods, further validating the decision to split the full sample.

Table 2 presents the pairwise Pearson correlations between the assets. Both Tables 1 and 2 are valuable for understanding the composition of the constructed VaR portfolios, as asset risk and their interrelationships are the primary factors influencing asset allocation. Notably, Table 2 shows that EUA has a very weak correlation with all metals, further supporting the idea of using metals for hedging purposes.

5. Results of minimum downside risk portfolios

5.1. Pre-crisis period

In this subsection, we present the findings for the multi-asset VaR portfolios constructed for the pre-crisis period. Table 3 outlines the optimized asset allocations, with the weights displayed to two decimal places to capture subtle differences between portfolios using various VaR models. It is clear that the variations in mathematical functions applied in VaR models have little effect on the portfolio composition, which holds significant practical importance. In other words, investors can transition between different VaR models without needing to alter their investment strategies.

Table 1 reveals that gold dominates the precious metals portfolio (PMP) with the largest allocation. This is primarily due to gold has the lowest risk (0.350) during the pre-crisis period. Although EUAs are the most volatile asset (1.217), they still maintain a notable allocation of around 9%. This can be attributed to the negative correlation between EUAs and gold (−0.077), the portfolio leading asset. Despite palladium being riskier than platinum (0.725 vs. 0.515), it has a slightly larger share. This could be because palladium has lower correlation with gold (0.260) compared to platinum correlation (0.525). Silver, on the other hand, is entirely excluded from the portfolio due to its relatively high risk (0.600) and strong correlation with gold (0.788).

In the industrial metals portfolio (IMP), aluminium holds the largest share, over 48%, due to its status as the least risky asset (0.480). Copper comes next with a share of over 25%, followed by lead, which accounts for almost 18%. The positions of copper and lead align with their respective levels of risk, the second lowest (0.540) and third lowest (0.615). Zinc has no share in the portfolio because of its relatively high risk (0.661) and, more importantly, its relatively high correlation with aluminium (0.400), the most dominant metal. Despite its very high risk (1.222), EUAs hold a share of over 8%. This is because EUAs have a very low correlation with all industrial metals, which justifies their relatively high share.

Panel B of Table 3 illustrates a gradual increase in the estimation of downside risk as we move from the standard parametric VaR model to the parametric CVaR model. Additionally, the downside risk for the IMP is slightly greater than that for the PMP. This aligns with the HEI findings, indicating that the PMP offers superior hedge effectiveness compared to the IMP. These results suggest that PMP provides a more effective hedge against extreme risk for EUAs.

The conclusion drawn in the previous paragraph, based on VaR and HEI levels, could be misleading since VaR alone does not indicate how accurately the theoretical VaR matches actual returns. Choosing the right VaR model is crucial for investors to avoid poor decisions and significant losses. As noted by Su et al. (2023), it is essential to assess all VaR model estimates against observed empirical returns. Consequently, this section conducts a back-test of theoretical VaR models using the Kupiec (1995) standard coverage test, with the results presented in

Table 2
Pairwise correlations between the assets in the two sub-periods.

Pre-crisis period	EUA	Gold	Silver	Platin.	Palladi.
EUA	1	−0.077	0.003	−0.001	0.020
Gold	−0.077	1	0.788	0.525	0.260
Silver	0.003	0.788	1	0.564	0.348
Platin.	−0.001	0.525	0.564	1	0.453
Palladi.	0.020	0.260	0.348	0.453	1
Crisis period	**EUA**	**Gold**	**Silver**	**Platin.**	**Palladi.**
EUA	1	0.061	0.106	0.126	0.102
Gold	0.061	1	0.774	0.531	0.377
Silver	0.106	0.774	1	0.613	0.458
Platin.	0.126	0.531	0.613	1	0.576
Palladi.	0.102	0.377	0.458	0.576	1

Pre-crisis period	EUA	Alumi.	Copper	Lead	Zinc
EUA	1	0.101	0.084	0.041	0.067
Alumi.	0.101	1	0.433	0.347	0.400
Copper	0.084	0.433	1	0.521	0.586
Lead	0.041	0.347	0.521	1	0.626
Zinc	0.067	0.400	0.586	0.626	1
Crisis period	**EUA**	**Alumi.**	**Copper**	**Lead**	**Zinc**
EUA	1	0.038	0.166	0.039	0.116
Alumi.	0.038	1	0.537	0.339	0.536
Copper	0.166	0.537	1	0.380	0.556
Lead	0.039	0.339	0.380	1	0.462
Zinc	0.116	0.536	0.556	0.462	1

Table 3
Structure of the minimum downside risk portfolios in the pre-crisis period.

PMP	Different distribution functions				IMP	Different distribution functions			
	Norm.	Log.	Hyps.	CVaR		Norm.	Log.	Hyps.	Lap.
Panel A: Portfolio structure									
EUA	8.97	8.93	8.92	8.92	EUA	8.18	8.14	8.13	8.13
Gold	77.25	77.27	77.28	77.28	Aluminium	48.63	48.65	48.66	48.66
Silver	0.00	0.00	0.00	0.00	Copper	25.30	25.32	25.32	25.32
Platinum	6.36	6.37	6.37	6.37	Lead	17.89	17.89	17.89	17.89
Palladium	7.42	7.43	7.43	7.43	Zinc	0.00	0.00	0.00	0.00
Σ	100	100	100	100	Σ	100	100	100	100
Panel B: Estimated downside risk and hedge effectiveness									
Downside risk	−0.742	−0.811	−0.844	−0.855	Downside risk	−0.940	−1.027	−1.069	−1.077
HEI	0.734	0.734	0.734	0.734	HEI	0.665	0.665	0.665	0.665

Note: All shares of assets in Table are in percent. PMP (IMP) stands for precious metals portfolio (industrial metals portfolio).

Table 4
Downside risk back-testing of the EUA portfolios in the pre-crisis period.

	Precious metals portfolio				Industrial metals portfolio			
	Norm.	Log.	Hyps.	CVaR	Norm.	Log.	Hyps.	CVaR
N	26	19	17	17	14	9	6	5
Z-score	3.755	1.780	1.216	1.216	0.441	−0.983	−1.837	−2.122
Prob.	0.000	0.075	**0.224**	**0.224**	**0.659**	0.326	0.066	0.034

Notes: N represents the count of violations. The bolded values highlight the model with the highest probability and the best performance in terms of adequacy.

Table 4.

Table 4 presents the number of violations of theoretical VaR models, along with a Z-score and a probability metric that assess the model adequacy. A higher probability value signifies a more reliable model. The Z-score can be either positive or negative. A negative Z-score indicates that fewer observed returns surpass the theoretical VaR, suggesting an overestimation of risk. On the other hand, a positive Z-score reveals that more observed returns exceed the theoretical VaR, implying an underestimation of risk. The closer the Z-score is to zero, the more accurately the theoretical model reflects the actual extreme risk.

The findings indicate that the hyper-secant VaR with higher kurtosis and the conditional VaR both exhibit the same level of violations concerning empirical downside risk. This results in identical Z-scores and probabilities, approximately 22%. In contrast, the classical VaR model outperforms the other downside risk models in the IMP portfolio, with a probability exceeding 65%. This suggests that while the IMP provides a slightly less effective hedge than the PMP, it is more accurate in aligning with actual empirical returns.

5.2. Crisis period

Table 5 reveals that the composition of the optimal portfolios changes drastically during the crisis period, likely due to deteriorating market conditions. In the PMP, only gold and EUAs remain in the portfolio, with gold share increasing at the expense of platinum and palladium. Gold now comprises nearly 90% of the portfolio due to its lower risk, while EUAs account for just over 10%. Despite EUAs being highly risky (1.229), their inclusion is justified by their very low correlation with the most dominant gold (0.061).

In contrast, lead emerges as the primary asset in the IMP portfolio, accounting for over 38%, with aluminium coming in second at over 31%. This shift occurs because lead has the lowest risk (0.603) during the crisis period, followed closely by aluminium (0.613). Copper is positioned third due to its third-highest risk (0.627), while zinc has a negligible share because it carries the highest risk among industrial metals. The low correlation of EUAs again influences their portfolio share. Despite their very high risk (1.234), EUAs are included only due to their minimal correlation with other industrial metals.

The low correlation between metals and EUAs appears to play a crucial role in portfolio structure. Several key factors explain why this correlation is so low. First, different market drivers – metal prices are influenced by industrial demand, global economic conditions, supply constraints, and mining output, whereas EUA prices are driven by regulatory policies, carbon reduction targets, energy sector trends, and geopolitical considerations related to climate change. Second, divergence in economic sensitivities – metals, particularly industrial metals, are closely tied to industrial production and economic cycles. In contrast, EUAs are primarily affected by environmental policy changes

Table 5
Structure of the minimum downside risk portfolios in the crisis period.

PMP	Different distribution functions				IMP	Different distribution functions			
	Norm.	Log.	Hyps.	CVaR		Norm.	Log.	Hyps.	Lap.
Panel A: Portfolio structure									
EUA	10.61	10.58	10.57	10.52	EUA	11.20	11.17	11.15	11.08
Gold	89.39	89.42	89.43	89.48	Aluminium	31.61	31.61	31.62	31.62
Silver	0.00	0.00	0.00	0.00	Copper	18.85	18.85	18.85	18.90
Platinum	0.00	0.00	0.00	0.00	Lead	38.24	38.24	38.25	38.26
Palladium	0.00	0.00	0.00	0.00	Zinc	0.12	0.13	0.13	0.14
Σ	100	100	100	100	Σ	100	100	100	100
Panel B: Estimated downside risk and hedge effectiveness									
VaR (1%)	−0.997	−1.089	−1.133	−1.142	VaR (1%)	−1.061	−1.159	−1.207	−1.219
HEI (1%)	0.647	0.647	0.648	0.648	HEI (1%)	0.626	0.626	0.626	0.627

and compliance with emission caps, which are less sensitive to short-term economic fluctuations. Third, different investment purposes – investors often view metals as physical assets or hedges against inflation and economic instability, while EU allowances serve mainly as compliance tools for companies subject to carbon regulations. Consequently, their demand is largely dictated by policy requirements rather than market speculation or traditional economic trends.

Table 6 evaluates the effectiveness of downside risk models, revealing a notable shift from the pre-crisis period. Specifically, the parametric CVaR now leads in the PMP, achieving an almost perfect match with a probability close to one. For the industrial metals portfolio, the logistic VaR performs the best, with a score exceeding 80%.

During the crisis period, precious metals provide a marginally better hedge against extreme risk, as shown in Table 5, while the CVaR measure aligns most accurately with empirical returns. This means that the PMP is superior in both categories relevant for investors when selecting the best portfolio.

Examining both the pre-crisis and crisis sub-periods, the precious metals portfolio demonstrates better hedging performance than the industrial metals portfolio, with the dominant role of gold being particularly evident. These results are very well in line with the existing papers, such as Baur and Lucey (2010), Vieira et al. (2023), Al-Nassar et al. (2023).

5.3. Forecasting

Forecasting plays a crucial role in research by enhancing the robustness of VaR models, aiding investors in managing risk exposure, and ensuring efficient capital allocation. This enables informed decision-making aligned with individual risk tolerance levels. Following the methodology of Chai and Zhou (2018), we apply the Kupiec (1995) test to assess the predictive accuracy of each VaR model. For this evaluation, both the pre-crisis and crisis periods are divided into in-sample and out-of-sample segments. The in-sample period for the pre-crisis phase spans from January 2015 to December 2017, while the out-of-sample period covers January 2018 to December 2019. During the crisis phase, the in-sample period includes January 2020 to December 2020, with the out-of-sample extending until August 2024.

Table 7 presents the VaR forecasting results. In the pre-crisis period, the forecasting performance of VaR models for the precious metals portfolio is notably weak, as the estimated probabilities are quite low. The hyper-secant VaR and CVaR models perform slightly better, with a probability of 0.088, though this is still insufficient for reliable forecasting. In contrast, the industrial metals portfolio shows strong forecasting results across all VaR models, with high probabilities indicating greater accuracy. The classical VaR model, in particular, stands out with a Z-score near zero and a probability close to one, marking it as the most accurate.

In the crisis period, however, the dynamics shift, with the precious metals portfolio outperforming the industrial metals portfolio in forecasting accuracy. This time, the hyper-secant and CVaR models show superior performance, achieving a much higher probability of 0.750, indicating strong forecasting precision. Conversely, within the industrial metals portfolio, while the classical VaR model still delivers the best result, its probability drops significantly to 0.268, highlighting its comparative underperformance relative to the precious metals portfolio.

Notably, the dominance of a particular portfolio remains consistent when considering both back-testing and forecasting assessments. Specifically, the industrial metals portfolio demonstrated stronger back-testing results during the pre-crisis period, whereas the precious metals portfolio took the lead during the crisis period. This consistency across different analyses underscores the robustness of the VaR models.

6. Omega ratio results

6.1. Omega ratio of individual assets

This section examines the Omega ratios of the selected assets, analysing them across five different threshold levels and two distinct sub-samples. This analysis is crucial for understanding the portfolio structure, as assets with higher Omega ratios tend to occupy a larger proportion of the portfolio. Table 8 details the Omega ratios for all commodities, while Fig. 2 illustrates the corresponding Omega functions in two sub-periods. The shape of the Omega function varies with different threshold levels, which reflect the risk tolerance (Avouyi-Dovi et al., 2004).

Table 8 reveals that most Omega ratios hover around one, where an Omega ratio above one signals a stronger outperformance relative to downside risk, and vice-versa. Additionally, as the threshold level increases, the Omega ratio tends to decrease, implying reduced potential for outperformance. In calculating Omega, there is a clear relationship between an asset daily returns, its variance and its Omega value. Essentially, assets with higher returns and lower risk generally exhibit higher Omega ratios, while the opposite is true for those with lower returns and higher risk.

During both periods, EUAs exhibit the highest Omega, despite being the most volatile asset. This elevated Omega is driven by the substantial growth recorded by EUAs, which compensates for their high risk. Apart from EUAs, palladium also consistently has an Omega above one across all threshold levels in the pre-crisis period. Although palladium is the riskiest precious metal during this time, its relatively high average returns (0.029) contribute to its elevated Omega. Gold, lead, and zinc achieve Omega values above one at lower threshold levels before the crisis, suggesting they may be suitable for inclusion in an Omega-based portfolio. In contrast, during the crisis period, gold, silver, aluminium, and copper demonstrate Omega values above one, largely due to their strong growth rates, as shown in Table 1.

Fig. 2 illustrates the Omega functions for the selected assets, providing a visual analysis that complements the results in Table 7. Unlike the table, the figure allows for an examination of the Omega function slopes. According to Botha (2007), the shape of the Omega function offers two key insights. A more gradual decline of the Omega function suggests a higher potential for positive returns, while a steeper slope indicates lower risk. As expected, all Omega functions exhibit a downward trend and are ranked according to their Omega values, with EUAs positioned at the top. In both graphs, the red line, representing gold, has the steepest slope, signalling the lowest risk, which aligns with the findings in Table 1.

6.2. Construction of the Omega portfolios

This section outlines the outcomes of the Omega portfolios that were

Table 6
Downside risk back-testing of the EUA portfolios in the crisis period.

	Precious metals portfolio				Industrial metals portfolio			
	Norm.	Log.	Hyps.	CVaR	Norm.	Log.	Hyps.	CVaR
N	21	14	13	12	17	11	9	9
Z-score	2.583	0.558	0.269	−0.020	1.504	−0.248	−0.832	−0.832
Prob.	0.010	0.577	0.788	**0.983**	0.133	**0.804**	0.405	0.405

Note: See Table 4.

Table 7
Model forecasting.

	Precious metals portfolio				Industrial metals portfolio			
	Norm.	Log.	Hyps.	CVaR	Norm.	Log.	Hyps.	CVaR
Panel A: Pre-crisis period								
N	13	10	9	9	5	4	3	3
Z-score	3.477	2.148	1.705	1.705	−0.009	−0.458	−0.906	−0.906
Prob.	0.001	0.032	**0.088**	**0.088**	**0.993**	0.647	0.365	0.365
Panel B: Crisis period								
N	7	6	5	5	2	1	0	0
Z-score	1.283	0.801	0.318	0.318	−1.268	−1.593	−2.079	−2.079
Prob.	0.199	0.423	**0.750**	**0.750**	**0.268**	0.111	0.038	0.038

Note: See Table 4.

Table 8
Omega ratios of the selected commodities.

Panel A: Pre-crisis period									
	EUA	Gold	Silver	Platinum	Palladium	Aluminium	Copper	Lead	Zinc
$\tau = 0.000$	1.112	1.028	0.985	0.936	1.047	1.005	0.987	1.010	1.018
$\tau = 0.002$	1.107	1.012	0.976	0.925	1.039	0.994	0.978	1.002	1.010
$\tau = 0.004$	1.102	0.997	0.967	0.914	1.032	0.984	0.969	0.993	1.002
$\tau = 0.006$	1.097	0.982	0.958	0.904	1.024	0.973	0.959	0.985	0.995
$\tau = 0.008$	1.092	0.967	0.949	0.893	1.016	0.963	0.950	0.977	0.987
Panel B: Crisis period									
	EUA	Gold	Silver	Platinum	Palladium	Aluminium	Copper	Lead	Zinc
$\tau = 0.000$	1.083	1.077	1.045	1.005	0.967	1.038	1.050	1.017	1.013
$\tau = 0.002$	1.078	1.064	1.039	0.999	0.963	1.029	1.042	1.009	1.006
$\tau = 0.004$	1.073	1.051	1.032	0.993	0.958	1.021	1.033	1.000	0.999
$\tau = 0.006$	1.068	1.038	1.026	0.986	0.953	1.012	1.025	0.992	0.991
$\tau = 0.008$	1.064	1.025	1.020	0.980	0.948	1.004	1.016	0.983	0.984

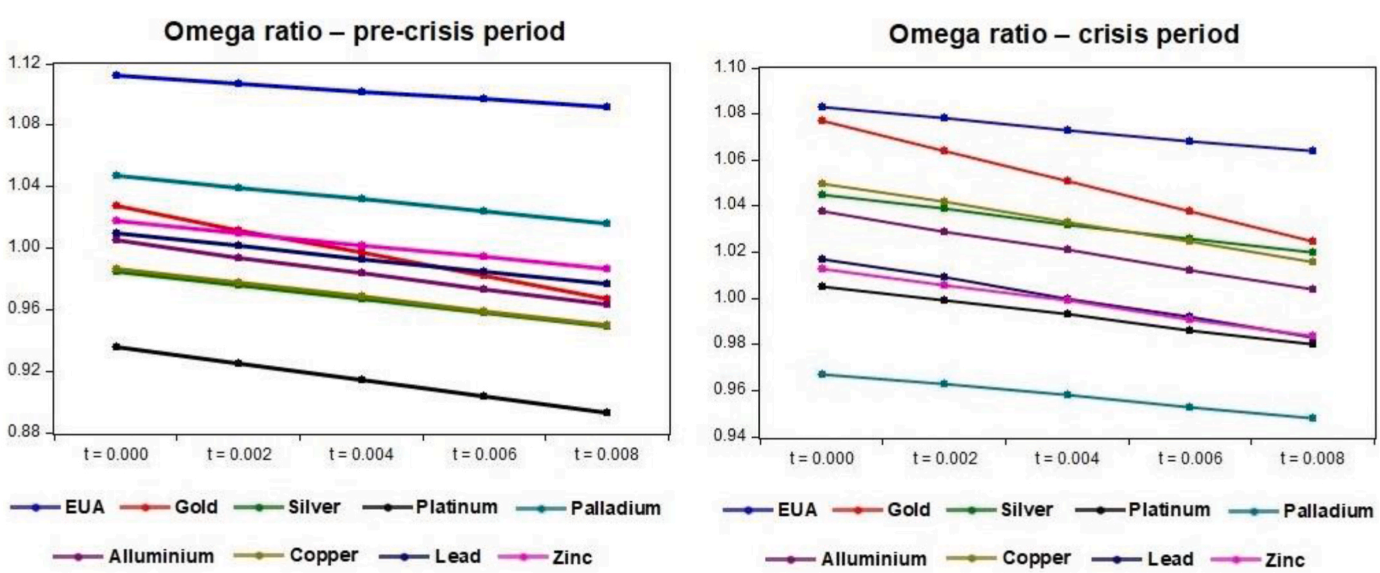

Fig. 2. Omega functions of the selected assets.

constructed, with the findings displayed in Table 9. These portfolios were developed across five different threshold levels, revealing variations in portfolio composition depending on the threshold applied. Insights from the previous chapter are valuable here, as the Omega ratio of each asset plays a crucial role in determining its proportion within the portfolio.

In the pre-crisis portfolio that includes precious metals, EUAs hold the largest share, and their proportion increases as the threshold level rises. This is mainly due to EUAs high daily returns, which give them the highest likelihood of surpassing the threshold and the lowest chance of significant negative returns. Conversely, palladium is the only precious metal included in the pre-crisis Omega portfolio. However, unlike EUAs, palladium share decreases as the threshold level increases, owing to its comparatively lower average returns. The other precious metals are excluded from the Omega portfolio because of their relatively low Omega ratios.

In the portfolio comprising industrial metals, EUAs also take the lead, beginning with an 80% share at the lowest threshold and expanding to

Table 9
Structure of the Omega portfolios in the pre-crisis period.

Precious metals portfolio						Industrial metals portfolio					
	Tau						Tau				
	0.000	0.002	0.004	0.006	0.008	Tau	0.000	0.002	0.004	0.006	0.008
EUA	68.83	70.71	76.38	81.07	89.84	EUA	80.57	86.78	92.53	100.00	100.00
Gold	0.00	0.00	0.00	0.00	0.00	Aluminium	0.00	0.00	0.00	0.00	0.00
Silver	0.00	0.00	0.00	0.00	0.00	Copper	0.00	0.00	0.00	0.00	0.00
Platinum	0.00	0.00	0.00	0.00	0.00	Lead	0.00	0.00	0.00	0.00	0.00
Palladium	31.17	29.29	23.62	18.93	13.16	Zinc	19.43	13.22	7.47	0.00	0.00
Σ	100	100	100	100	100	Σ	100	100	100	100	100

100% at the two highest thresholds. Similar to the precious metals portfolio, only one industrial metal (zinc) is included. Zinc inclusion is due to its having the highest Omega ratio across the various thresholds. The other industrial metals are excluded from the portfolio.

The composition of the Omega portfolios shifts during the crisis period, as detailed in Table 10. Notably, gold and EUAs become the primary components of the portfolio. Gold begins with a roughly 65% share, which decreases as the threshold increases, while EUAs start at around 35%, with their share growing as the threshold rises. The low risk of gold provides it with a strong initial position, but as the threshold increases, the higher daily returns of EUAs become more dominant, compensating for their increased risk. Other precious metals are not the part of the portfolio in the crisis period.

In the industrial metals portfolio, EUAs are joined by aluminium and lead at the lowest threshold levels. Aluminium is included due to its relatively high Omega, driven by strong average returns, while lead is selected for its relatively low risk. However, as the threshold increases, the superior daily returns of EUAs push the industrial metals out of the portfolio at the highest threshold levels.

Table 11 displays the Omega results for the portfolios, revealing that the portfolio Omega values exceed those of any single asset, demonstrating the effectiveness of the optimizations. Across all threshold levels and in both sub-periods, the precious metals portfolio consistently outperforms the industrial metals portfolio. This suggests that the precious metals portfolio offers a better return-to-risk ratio.

Fig. 3 provides a side-by-side comparison of the 2 Ω functions. The blue line, representing the precious metals portfolio, is notably higher than the red line, which indicates the industrial metals portfolio. This clearly illustrates the superior Omega ratio of the precious metals portfolio. Additionally, a closer examination reveals that the blue line has a slightly steeper slope compared to the red line, suggesting that the precious metals portfolio carries lower risk.

Table 12 provides an explanation for the findings in Fig. 3 by presenting the first two moments at the lowest threshold level. While the precious metals portfolio shows a slightly higher mean return compared to the industrial metals portfolio, it also has substantially lower risk. This combination accounts for the higher Omega ratio of the precious metals portfolio and the steeper slope observed in its Omega function.

7. Discussion

This study presents a wealth of findings, as the research is conducted from multiple perspectives. First, the analysis shows that both precious and industrial metals can serve as effective hedges against extreme risk in EUAs. The dominance of gold in the portfolio is confirmed in both subsamples, which is consistent with existing literature (AlKhazali et al., 2021; Alomari et al., 2022; Wang et al., 2022b). However, back-testing analysis determines that different portfolios perform better in specific subsamples. For example, the industrial metals portfolio yields better results than precious metals in the pre-crisis period. This finding aligns with Adekoya and Oliyide (2020) and Živkov et al. (2024), who stated that industrial metals can successfully hedge against oil shocks. On the other hand, the precious metals portfolio performs better during the crisis period. Identifying the best risk-minimizing portfolio is crucial because it helps protect against significant losses, thereby preserving capital over time. This is particularly important during market downturns or periods of high volatility, where poorly managed portfolios can suffer substantial declines.

The study assesses downside risk through various VaR models, revealing subtle differences in the extreme risk estimates. It becomes clear that there is not a one-size-fits-all solution when it comes to selecting the best VaR model. Both high-kurtosis and low-kurtosis VaR models prove valuable under specific conditions. Therefore, investors need to continuously evaluate which model is most appropriate given the portfolio composition and prevailing market conditions. This ongoing assessment is vital because an effective VaR model delivers more precise risk estimates, allowing for better anticipation of potential losses. Furthermore, a reliable VaR model instils confidence in decision-makers, enabling more strategic and informed choices regarding investments, asset allocations, and risk management strategies.

Additionally, the study seeks to identify the optimal Omega portfolio, which plays a significant role in extreme risk analysis by measuring the probability-weighted returns above a certain threshold. This

Table 11
Omega ratios of the two portfolios.

Period	Portfolio with metals	Threshold levels				
		τ = 0.000	τ = 0.002	τ = 0.004	τ = 0.006	τ = 0.008
Pre-crisis	Precious	1.135	1.128	1.121	1.114	1.108
	Industrial	1.122	1.116	1.110	1.105	1.100
Crisis	Precious	1.133	1.223	1.113	1.104	1.096
	Industrial	1.111	1.104	1.098	1.092	1.088

Table 10
Structure of the Omega portfolios in the crisis period.

	Precious metals portfolio						Industrial metals portfolio				
Tau	0.000	0.002	0.004	0.006	0.008	Tau	0.000	0.002	0.004	0.006	0.008
EUA	35.45	41.22	46.62	53.47	62.00	EUA	60.95	71.47	81.71	97.44	100.00
Gold	64.55	58.78	53.38	46.53	38.00	Aluminium	27.90	24.16	18.29	2.56	0.00
Silver	0.00	0.00	0.00	0.00	0.00	Copper	0.00	0.00	0.00	0.00	0.00
Platinum	0.00	0.00	0.00	0.00	0.00	Lead	11.15	4.37	0.00	0.00	0.00
Palladium	0.00	0.00	0.00	0.00	0.00	Zinc	0.00	0.00	0.00	0.00	0.00
Σ	100	100	100	100	100	Σ	100	100	100	100	100

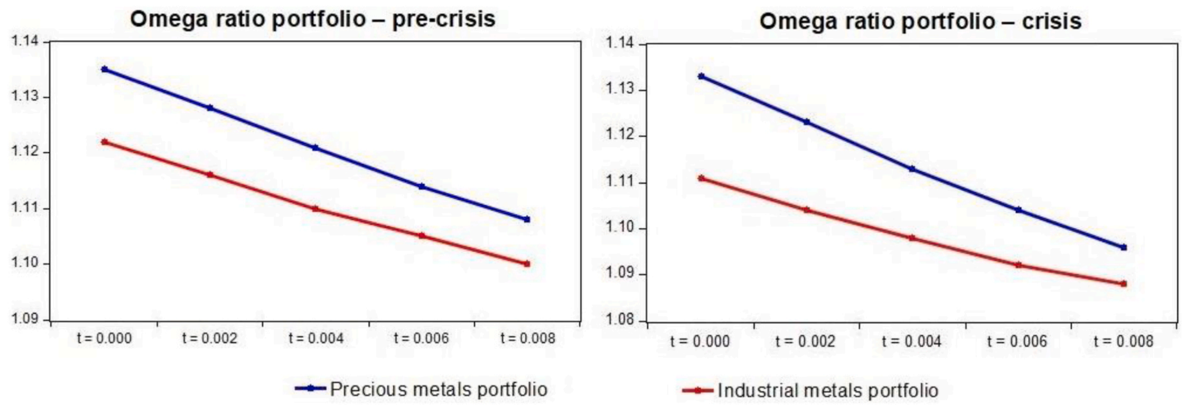

Fig. 3. Omega functions of the two portfolios.

Table 12
First two moments of the 0% threshold Omega portfolios.

	Pre-crisis period		Crisis period	
	Precious	Industrial	Precious	Industrial
Mean	0.045	0.043	0.024	0.023
Variance	1.160	1.511	0.294	0.623

approach provides a thorough understanding of both the upside potential and downside risk within a portfolio. The findings suggest that the precious metals portfolio outperforms the industrial metals portfolio across all threshold levels, indicating that investors can achieve superior risk-adjusted returns by combining EUAs with precious metals. The Omega ratio sensitivity to the tails of the return distribution is particularly valuable in this research, as it addresses extreme losses that can severely impact portfolio performance. The precious metals portfolio, where gold is the only auxiliary asset, effectively minimizes exposure to these risks while still offering the potential for gains.

8. Conclusion

This paper constructs two five-asset portfolios composed of EUAs and metals precious, aiming to determine which portfolio has a lower exposure to extreme risk and a better return-to-risk profile. Extreme risk is measured using parametric VaR models, including the classical normal VaR, two non-normal VaR models and CVaR model. On the other hand, the Omega ratio is used as a performance measure for return-to-risk results. Both portfolios are constructed for the pre-crisis and crisis periods to evaluate which portfolio performs better across two distinctively different subsamples.

The structure of all constructed VaR portfolios is quite similar, suggesting that variations in objective functions have little effect on the portfolio configuration. However, the choice of VaR function does influence the magnitude of the estimated downside risk, which is critical for ensuring the accuracy of the VaR model and the precise evaluation of extreme risk. Back-testing indicates that while the normal VaR model for the industrial metals portfolio outperforms the others, its performance significantly deteriorates during the crisis period. Conversely, the CVaR estimated downside risk closely aligns with actual returns in the precious metals portfolio. The superiority of certain portfolios is consistent when considering both back-testing and forecasting results. Both portfolios serve as effective hedges for EUAs, achieving an extreme risk reduction of over 60% in both sub-periods. However, the precious metals portfolio, where gold plays a dominant role, performs slightly better than the industrial metals portfolio. This finding is consistent with the existing literature.

Analysing the Omega ratio, the precious metals portfolio consistently outperforms the industrial metals portfolio at every threshold level, suggesting that investors can attain better risk-adjusted returns by pairing EUAs with precious metals. This advantage is largely due to gold, which exhibits significantly lower risk compared to other metal commodities.

This paper offers valuable insights for market participants in EUA futures market by delivering several key takeaways. Firstly, it emphasizes the importance of testing multiple VaR models before making investment decisions. Secondly, it clearly identifies the optimal investment choices by evaluating extreme risk levels and return-to-risk outcomes. Thirdly, the notably different results between the two subsamples indicate that VaR and Omega portfolios should be recalculated regularly, as their composition is influenced by ever-changing market conditions. Overall, these findings provide important guidance for investors as well as policymakers. Investors can better evaluate and manage carbon market risk by taking positions in futures contracts that counterbalance their exposures. At the same time, policymakers can improve market stability by tracking price trends and risk variations, enabling them to implement more targeted and effective interventions in the carbon markets.

CRediT authorship contribution statement

Dejan Živkov: Writing – original draft, Software, Methodology, Data curation, Conceptualization. **Boris Kuzman:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Methodology, Writing – original draft, Writing – review & editing. **Miloš Japundžić:** Conceptualization, Data curation, Formal analysis, Methodology, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors have nothing to declare.

Acknowledgments

The paper is a part of research financed by the MSTDI RS, agreed in Decision No. 451-03-66/2024-03/200009 from February 5, 2024.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.resourpol.2024.105447.

Data availability

We submitt our working files that show calculations were done.

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